

Flavor Changing Processes in Supersymmetric Models with Hybrid Gauge- and Gravity-Mediation

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Abstract

We consider supersymmetric models where gauge mediation provides the dominant contributions to the soft supersymmetry breaking terms while gravity mediation provides sub-dominant yet non-negligible contributions. We further assume that the gravity-mediated contributions are subject to selection rules that follow from a Froggatt-Nielsen symmetry. This class of models constitutes an example of viable and natural non-minimally flavor violating models. The constraints from $K^0 - \bar{K}^0$ mixing imply that the modifications to the Standard Model predictions for $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing are generically at most at the percent level, but can be of order ten percent for large $\tan \beta$. The modifications for $D^0 - \bar{D}^0$ mixing are generically at most of order a few percent, but in a special subclass of models they can be of order one. We point out $\Delta B = 1$ processes relevant for flavor violation in hybrid mediation.

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I. INTRODUCTION

The physics of flavor and CP violation could be rich with deviations from the Standard Model predictions if supersymmetry is realized at the TeV scale, and if the mechanism that mediates its breaking to the minimal supersymmetric standard model (MSSM) is not minimally flavor violating (MFV). Indeed, hybrid models of gauge- and gravity-mediation can lead to flavor violating effects large enough to be explored by the LHC experiment [1, 2]. It is the purpose of this work to study whether the flavor- and CP-violating effects expected in this framework can be discovered in decays of D , B_d and B_s mesons.

The basic idea of the hybrid gauge-gravity models is the following. There are gauge-mediated contributions to the soft supersymmetry breaking terms. If the scale of gauge mediation is low, then the gravity-mediated contributions are negligible, and the model is MFV. If the scale is very high, then the gravity-mediated contributions dominate, and it requires a very careful model building to suppress the supersymmetric contributions to flavor changing neutral current (FCNC) processes [3]. There is, however, an intermediate range for the scale of gauge mediation where the gravity-mediated contributions are neither negligible nor dominant [4]. For this range of scales, the Froggatt-Nielsen (FN) mechanism [5] – an approximate horizontal Abelian symmetry – can play a role in suppressing the squark and slepton mixing in a simple and natural way [6, 7].

If the ATLAS/CMS experiments can measure the mass splitting between squarks or sleptons, we will learn about the relative importance of the gauge- and gravity-mediated contributions and thereby on the gauge mediation scale. If these experiments can measure the flavor decomposition of squarks and sleptons, the FN framework can be tested and we may further learn about the way that the FN symmetry is implemented. Here we would like to ask whether the (present and future) B -factories, the TeVatron experiments and the LHCb experiment can give early hints to this framework, before direct squark and slepton measurements are achieved.

The paper is organized as follows. We review FCNC constraints in SUSY models in Section II. In Section III we work out the flavor-violating low energy couplings in the hybrid gauge-gravity models, and discuss their phenomenology in view of FCNC data in Section IV. Section V contains the phenomenological consequences of a variant of FN models with holomorphic zeros. In Section VI we discuss general properties of gauge mediation in the

context of flavor constraints and comment on hidden sector effects. We conclude in Section VII. The Appendix contains details on the effects of MSSM renormalization group running.

II. FCNC CONSTRAINTS ON SUSY PARAMETERS

New physics at the TeV scale could lead to enhancement of FCNC processes by orders of magnitude. The fact that such an enhancement has not been observed in any of the $s \rightarrow d$, $c \rightarrow u$, $b \rightarrow d$ and $b \rightarrow s$ transitions gives strong constraints on the flavor structure of the new physics. We discuss constraints on SUSY parameters from gluino loops in Section IIA and from chargino contributions in Section IIB. The impact of rare decays and the constraints that arise at large $\tan\beta$ are covered in Section IIC.

A. Gluino contributions

In the supersymmetric framework, the following combinations of parameters are strongly constrained by processes involving $q_i \rightarrow q_j$ transitions:

$$\delta_{ij}^q = \frac{1}{\tilde{m}_q^2} \sum_{\alpha} K_{i\alpha}^q K_{j\alpha}^{q*} \Delta \tilde{m}_{q\alpha}^2. \quad (2.1)$$

Here $K_{i\alpha}^q$ is the mixing angle in the coupling of the gluino (and similarly the bino and neutral wino) to $q_i - \tilde{q}_\alpha$, $\tilde{m}_q^2 = \frac{1}{3} \sum_{\alpha=1}^3 \tilde{m}_{q\alpha}^2$ is the average squark mass-squared, and $\Delta \tilde{m}_{q\alpha}^2 = \tilde{m}_{q\alpha}^2 - \tilde{m}_q^2$. Using the unitarity of the mixing matrix K , we can write

$$\tilde{m}_q^2 \delta_{ij}^q = \sum_{\alpha} K_{i\alpha}^q K_{j\alpha}^{q*} (\Delta \tilde{m}_{q\alpha}^2 + \tilde{m}_q^2) = (\tilde{M}_q^2)_{ij}, \quad (2.2)$$

where \tilde{M}_q^2 is the mass-squared matrix for the squarks \tilde{q} in the basis where the quark q masses and the gluino couplings are diagonal.

The mass-squared matrices carry also chirality indices, $M, N = L, R$, *i.e.* $(\tilde{M}_q^2)_{ij}^{MN}$ is the $\tilde{q}_{Mi}^\dagger \tilde{q}_{Nj}$ mass-squared term. Correspondingly, the δ_{ij}^q are assigned chirality indices, namely the FCNC constrain $(\delta_{ij}^q)_{MN}$. In the case that the $\tilde{q}_L - \tilde{q}_R$ mixing can be neglected, there are four classes of $(\delta_{ij}^q)_M \equiv (\delta_{ij}^q)_{MM}$: $(\delta_{ij}^d)_L$ for the left-handed down squarks \tilde{D}_L , $(\delta_{ij}^u)_L$ for the left-handed up squarks \tilde{U}_L , $(\delta_{ij}^d)_R$ for the right-handed down squarks \tilde{D}_R , and $(\delta_{ij}^u)_R$ for the right-handed up squarks \tilde{U}_R . We also define

$$\langle \delta_{ij}^q \rangle = \sqrt{(\delta_{ij}^q)_L (\delta_{ij}^q)_R}. \quad (2.3)$$

TABLE I: The phenomenological upper bounds on $(\delta_{ij}^q)_A$ and on $\langle \delta_{ij}^q \rangle$, where $q = u, d$ and $A = L, R$. The constraints are given for $m_{\tilde{q}} = 1$ TeV and $x \equiv m_{\tilde{g}}^2/m_{\tilde{q}}^2 = 1$. We assume that the phases could suppress the imaginary parts by a factor ~ 0.3 . The bound on $(\delta_{23}^d)_R$ is about 3 times weaker than that on $(\delta_{23}^d)_L$ (given in table). The constraints on $(\delta_{12,13}^d)_A$, $(\delta_{12}^u)_A$ and $(\delta_{23}^d)_A$ are based on, respectively, Refs. [9], [10] and [11].

q	ij	$(\delta_{ij}^q)_A$	$\langle \delta_{ij}^q \rangle$
d	12	0.03	0.002
d	13	0.2	0.07
d	23	0.6	0.2
u	12	0.1	0.006

In some cases, a two generation effective framework is useful. To understand that, consider a case where (no summations over i, j, k):

$$\begin{aligned}
|K_{ik}K_{jk}^*| &\ll |K_{ij}K_{jj}^*|, \\
|K_{ik}K_{jk}^*\Delta\tilde{m}_{q_kq_i}^2| &\ll |K_{ij}K_{jj}^*\Delta\tilde{m}_{q_jq_i}^2|,
\end{aligned}
\tag{2.4}$$

where $\Delta\tilde{m}_{q_jq_i}^2 = \tilde{m}_{q_j}^2 - \tilde{m}_{q_i}^2$. Then the contribution of the intermediate \tilde{q}_k can be neglected and, furthermore, to a good approximation, $K_{ii}K_{ji}^* + K_{ij}K_{jj}^* = 0$. For these cases, we obtain

$$\delta_{ij}^q = \frac{\Delta\tilde{m}_{q_jq_i}^2}{\tilde{m}_q^2} K_{ij}^q K_{jj}^{q*}.
\tag{2.5}$$

It is further useful to use instead of \tilde{m}_q the following average mass scale [8]:

$$\tilde{m}_{ij}^q = \frac{1}{2}(\tilde{m}_{q_i} + \tilde{m}_{q_j}).
\tag{2.6}$$

Constraints of the form $\delta_{ij}^q \ll 1$ imply that either quasi-degeneracy ($\Delta\tilde{m}_{q_iq_j}^2 \ll \tilde{m}_{ij}^{q^2}$) or alignment ($|K_{ij}^q| \ll 1$) or a combination of the two mechanisms is at work. We use the constraints obtained in Refs. [9, 10, 11]. They are presented in Table I. Wherever relevant, we allow a mild phase suppression in the mixing amplitude, namely we quote the stronger between the bounds on $\mathcal{R}e(\delta_{ij}^q)$ and $3\mathcal{I}m(\delta_{ij}^q)$. We would like to emphasize the following points:

1. The bounds have a strong dependence on the average squark mass, scaling roughly as $m_{\tilde{q}}/(1 \text{ TeV})$.
2. The bounds have a milder dependence on the ratio $x \equiv m_{\tilde{g}}^2/m_{\tilde{q}}^2$. In particular, for $x = 4$, the bound on $(\delta_{12}^d)_A$ ($\langle \delta_{12}^d \rangle$) is weakened to 0.06 (0.003).
3. If we allow an arbitrarily strong suppression of the CP violating phases, some bounds are further relaxed. For example, with zero phase, $m_{\tilde{q}} = 1 \text{ TeV}$ and $x = 1$, we have $\langle \delta_{12}^d \rangle \leq 0.004$.
4. The bounds compiled in Table I are based on conservative estimates. At large $\tan \beta$ the bounds can be significantly stronger and are more model-dependent, see Section II C.

B. Chargino contributions

Chargino contributions could also be of interest. If $\tan \beta$ is not very large, then for the various processes of interest the charged higgsino contributions are suppressed by small Yukawa couplings. We focus then on the charged wino contributions to $d_i \rightarrow d_j$ transitions, which involve intermediate $\tilde{u}_{L\alpha}$ squarks. Now the following combination is constrained (we omit here the chirality index L):

$$\delta_{ij}^{cu} = \frac{1}{\tilde{m}_u^2} \sum_{\alpha} Z_{i\alpha}^u Z_{j\alpha}^{u*} \Delta \tilde{m}_{u\alpha}^2. \quad (2.7)$$

Here $Z_{i\alpha}^u$ is the mixing angle in the coupling of the wino to $d_i - \tilde{u}_{\alpha}$ (both ‘left-handed’). Note that

$$\tilde{m}_u^2 \delta_{ij}^{cu} = \sum_{\alpha} Z_{i\alpha}^u Z_{j\alpha}^{u*} (\Delta \tilde{m}_{u\alpha}^2 + \tilde{m}_u^2) = (\widetilde{M}_u^{c2})_{ij}, \quad (2.8)$$

where \widetilde{M}_u^{c2} is the mass-squared matrix for the left-handed up squarks \tilde{u}_L in the basis where the *down* quark masses and the gluino couplings are diagonal. Note that $\delta_{ij}^{cu} \neq \delta_{ij}^u$. In particular,

$$Z^u = V^{\dagger} K^u, \quad (2.9)$$

where V denotes the CKM quark mixing matrix.

Consider, for example, δ_{12}^{cu} and assume that the conditions for an effective 2-flavor framework, Eq. (2.4), hold. Then, defining $\sin \tilde{\theta}_u \equiv K_{12}^u$, we obtain

$$\tilde{m}_u^2 \delta_{12}^{cu} = \frac{1}{2} \sin(2\tilde{\theta}_u - 2\theta_c) \Delta \tilde{m}_{u_2 u_1}^2, \quad (2.10)$$

where θ_c denotes the Cabibbo angle. On the other hand,

$$\tilde{m}_u^2 \delta_{12}^u = \frac{1}{2} \sin(2\tilde{\theta}_u) \Delta \tilde{m}_{u_2 u_1}^2. \quad (2.11)$$

Given a bound on $(\delta_{ij}^d)_L$ from gluino loops, by $SU(2)$ symmetry there is a corresponding bound on δ_{ij}^{cu} (see Section IIIB for details). The latter is often stronger than the bound from chargino contributions by approximately a factor of (α_3/α_2) , though there is further dependence on the gaugino masses via known loop functions.

If the mixing angles are small, Eq. (2.10), in general involving arbitrary two generations, can be linearized and yields

$$\delta_{ij}^{cu} = \delta_{ij}^u + V_{ji}^* \frac{\Delta \tilde{m}_{u_j u_i}^2}{\tilde{m}_u^2}. \quad (2.12)$$

This decomposition is commonly used to constrain δ^u through chargino interactions in rare processes, *e.g.*, [12]. The separation into ‘flavor diagonal’ and δ^u -induced terms, however, is not useful in models where there are cancelations between K^u and the CKM matrix elements.

C. $\Delta B = 1$ processes and large $\tan \beta$

A multitude of $\Delta B = 1$ decay observables has been measured so far [13]. The most interesting ones for the purpose of constraining new physics parameters are those which have reasonable theoretical and experimental uncertainties, and depend only on a small set of model parameters. Given these requirements, very useful modes are radiative and (semi)-leptonic decays mediated by $b \rightarrow q\gamma$ and $b \rightarrow q\ell^+\ell^-$ for $\ell = e, \mu$ and $q = d, s$. Currently, theory gives preference to inclusive versus exclusive decays, although the purely leptonic and very rare $B \rightarrow \ell^+\ell^-$ decays are also important. Future data on dedicated distributions and asymmetries in FCNC exclusive decays, which will become available in the LHC era [11], will also be of relevance.

For the constraints on δ_{23}^d in Table I, data on $B \rightarrow X_s \ell^+ \ell^-$, $B \rightarrow X_s \gamma$ decays and B_s mixing has been employed. For the radiative and semileptonic $b \rightarrow d$ decays, the experi-

mental situation is currently not as good as for $b \rightarrow s$ decays, and only B_d mixing has been used to limit δ_{13}^d .

The impact of $\Delta B = 1$ versus $\Delta B = 2$ processes for the bounds on the δ^q parameters has a complex dependence on the model parameters. For example, for $(\delta_{23}^d)_R$, the strongest constraint comes from $\Delta B = 2$, whereas for $(\delta_{23}^d)_L$ and $\langle \delta_{23}^d \rangle$ the rare $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$ decays strengthen the bounds from meson mixing and, for some regions of the parameter space, even provide the best limits, see *e.g.* [14] for a study with small to moderate $\tan \beta$.

We now consider more model-dependent bounds arising for large $\tan \beta$, where also the $B \rightarrow \ell^+ \ell^-$ decays come into play. The dependence of the δ_{23}^d -bounds on $\tan \beta$ and SUSY mass terms can be seen, *e.g.*, in [15].

An important mechanism for $\tan \beta$ enhancements are Higgs penguins, magnifying gluino loops with down squark flavor mixing in $\Delta B = 1$, notably $B \rightarrow \mu^+ \mu^-$ decays, and $\Delta B = 2$ processes, *e.g.*, [16]. Using the recent 95% C.L. bounds on the branching ratios, $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-) < 1.8 \cdot 10^{-8}$ and $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) < 5.8 \cdot 10^{-8}$ [17], we obtain for $\tan \beta = 30$, $x = 1$ and $A = L, R$

$$|(\delta_{13}^d)_A| < 0.04 \cdot \left(\frac{M_{A^0}}{200 \text{ GeV}} \right)^2, \quad |(\delta_{23}^d)_A| < 0.06 \cdot \left(\frac{M_{A^0}}{200 \text{ GeV}} \right)^2, \quad (2.13)$$

where M_{A^0} denotes the pseudoscalar Higgs mass. The bounds scale very roughly as $(30/\tan \beta)^3$, and also depend via non-holomorphic corrections on the higgsino parameters. Since the experimental limits are a factor of ~ 10 (100) away from the corresponding Standard Model branching ratios for $B_s(B_d) \rightarrow \mu^+ \mu^-$ decays, the bound on δ_{23}^d is more constraining than the one on δ_{13}^d .

Bounds in a similar ballpark can be obtained from neutral Higgs exchange effects in B_s and B_d mixing (for $\tan \beta = 30, x = 1$) [16]:

$$\langle \delta_{13}^d \rangle < 0.01 \cdot \left(\frac{M_{A^0}}{200 \text{ GeV}} \right), \quad \langle \delta_{23}^d \rangle < 0.04 \cdot \left(\frac{M_{A^0}}{200 \text{ GeV}} \right), \quad (2.14)$$

which scale roughly as $(30/\tan \beta)^2$.

The constraints in Eq. (2.13) and Eq. (2.14) can be stronger than those given in Table I, but can be evaded by large M_{A^0} and by small $\tan \beta$. Note that the mixing bounds decouple slower than the $B \rightarrow \mu^+ \mu^-$ ones, so in order to have large effects in the rare decays, either a very large $\tan \beta$ or a very light Higgs is required, or a hierarchy between the $(\delta_{i3}^d)_L$ and $(\delta_{i3}^d)_R$ parameters such that $\langle \delta_{i3}^d \rangle$ is small.

III. HYBRID GAUGE-GRAVITY MEDIATION

Ref. [1] has considered a mediation mechanism that allows non-MFV contributions to the soft supersymmetry breaking terms, yet flavor changing terms are naturally suppressed. The basic assumption is that the gauge-mediated contributions are dominant, but gravity-mediated contributions are non-negligible. The structure of the latter is, however, not arbitrary. An approximate Abelian symmetry which explains the smallness and the hierarchy of the Yukawa couplings (the Froggatt-Nielsen mechanism) dictates at the same time a flavor structure for the soft terms.

In this Section, we analyze the predictions of this framework for the flavor changing δ_{ij}^q parameters. We write down the high scale soft terms in Section III A, and include effects from renormalization group evolution (RGE) in Section III B. Therein we also present the low energy δ^q parameters in hybrid mediation. Mass splittings and flavor mixing matrices are considered in Section III C.

A. Gauge and gravity soft breaking

The soft breaking terms for the squarks have then the following form, at the scale of gauge mediation, m_M :

$$\begin{aligned} M_{Q_L}^2(m_M) &= \tilde{m}_{Q_L}^2(\mathbf{1} + rX_{Q_L}), \\ M_{D_R}^2(m_M) &= \tilde{m}_{D_R}^2(\mathbf{1} + rX_{D_R}), \\ M_{U_R}^2(m_M) &= \tilde{m}_{U_R}^2(\mathbf{1} + rX_{U_R}), \end{aligned} \tag{3.1}$$

where $r \lesssim 1$ parameterizes the ratio between the gravity-mediated and the gauge-mediated contributions, and is discussed further in Section VI. While the gauge-mediated initial conditions are flavor blind, the structure of the X_{q_A} matrices, coming from gravity mediation, is subject to the selection rules of the FN symmetry.

The diagonal terms of the X_{q_A} matrices are never suppressed by the horizontal symmetry. On the other hand, the off-diagonal entries are suppressed whenever the two corresponding generations carry different H -charges. Within the simplest FN models, with a single horizontal $U(1)_H$ symmetry, the parametric suppression of the off-diagonal terms is related to

that of the quark parameters:

$$(X_{q_{L,R}})_{ii} \sim 1, \quad (X_{q_L})_{ij} \sim |V_{ij}|, \quad (X_{q_R})_{ij} \sim \frac{m_{q_i}/m_{q_j}}{|V_{ij}|} \quad (i < j), \quad q = U, D. \quad (3.2)$$

The “ \sim ” sign here means “of the same parametric suppression as” but with generally different $\mathcal{O}(1)$ complex coefficients.

The squark mass-squared matrices $\overline{M}_{\tilde{q}_A}^2$ then have the following form:

$$\begin{aligned} \overline{M}_{\tilde{D}_L}^2 &= M_{\tilde{Q}_L}^2 + D_{D_L} \mathbf{1} + m_D m_D^\dagger, \\ \overline{M}_{\tilde{U}_L}^2 &= M_{\tilde{Q}_L}^2 + D_{U_L} \mathbf{1} + m_U m_U^\dagger, \\ \overline{M}_{\tilde{D}_R}^2 &= M_{\tilde{D}_R}^2 + D_{D_R} \mathbf{1} + m_D^\dagger m_D, \\ \overline{M}_{\tilde{U}_R}^2 &= M_{\tilde{U}_R}^2 + D_{U_R} \mathbf{1} + m_U^\dagger m_U, \end{aligned} \quad (3.3)$$

where $m_{U,D}$ are the up and down quark mass matrices in the flavor basis, D_{q_A} are the D -term contributions and all quantities should be evaluated at the electroweak scale $\mu \sim m_Z$. We assume that $r > y_t^2 |V_{ts}|^2 \sim 0.002$, so that the gravity-mediated contributions are non-negligible.

B. Flavor breaking at m_Z

The initial conditions (3.1) hold at the scale of gauge mediation, m_M , and the flavor relations (3.2) hold at the scale of gravity mediation, the Planck mass m_{Pl} . We are, however, interested in the predictions for the $(\delta_{ij}^q)_A$ parameters, requiring soft terms evaluated at the electroweak scale. We thus need to take into account the effects of renormalization group evolution. A detailed discussion of the RGE is given in Appendix A. The final conclusions are the following:

(i) Starting from the soft squark masses at the scale m_M of the form given in Eq. (3.1), the soft squark masses at the scale m_Z can be written in the following approximate form:

$$\begin{aligned} M_{\tilde{Q}_L}^2(m_Z) &\sim \tilde{m}_{\tilde{Q}_L}^2(r_3 \mathbf{1} + c_u Y_u Y_u^\dagger + c_d Y_d Y_d^\dagger + r X_{Q_L}), \\ M_{\tilde{U}_R}^2(m_Z) &\sim \tilde{m}_{\tilde{U}_R}^2(r_3 \mathbf{1} + c_{uR} Y_u^\dagger Y_u + r X_{U_R}), \\ M_{\tilde{D}_R}^2(m_Z) &\sim \tilde{m}_{\tilde{D}_R}^2(r_3 \mathbf{1} + c_{dR} Y_d^\dagger Y_d + r X_{D_R}), \end{aligned} \quad (3.4)$$

where Y_u and Y_d denote the up and down quark Yukawa matrices in the flavor basis.

(ii) The relations between the off-diagonal elements $(X_{qL,R})_{ij}$ and the quark parameters, given in Eq. (3.2), are either RGE-invariant to a good approximation, or changed by factors of $\mathcal{O}(1)$. In any case, the relations between the parametric suppressions remain the same, and one should simply use the low energy values of $|V_{ij}|$ and of m_{q_i}/m_{q_j} to estimate the low energy values of $(X_{qL,R})_{ij}$.

(iii) We define the factor r_3 via the RGE correction to the diagonal elements of the soft squark mass matrices $(M_{\tilde{q}A}^2)_{ii}$:

$$\tilde{m}_{12}^2(\mu = m_Z) = r_3 \tilde{m}_{12}^2(\mu = m_M), \quad (3.5)$$

with the average diagonal mass-squared defined as

$$\tilde{m}_{ij}^2 \equiv \frac{1}{2} \left((M_{\tilde{q}A}^2)_{ii} + (M_{\tilde{q}A}^2)_{jj} \right). \quad (3.6)$$

In writing Eqs. (3.5) and (3.6) with the same \tilde{m}_{12}^2 and r_3 for all three sectors $(\tilde{Q}_L, \tilde{U}_R, \tilde{D}_R)$ we take into account that the dominant contribution to the initial squark soft masses and to their RGE is QCD-induced and, in the limit that we neglect the electroweak gauge couplings, is universal among all squarks. Numerically, r_3 is of $\mathcal{O}(1 - 10)$, depending on the initial conditions and the scale of supersymmetry breaking. Details on r_3 in gauge mediation are given in Section VI. In minimal models, typically $r_3 \sim 3$.

(iv) The coefficients c_u, c_d, c_{uR}, c_{dR} are of order $[5/(16\pi^2)] \ln(m_M/m_Z)$ and can be $\mathcal{O}(1)$ for $m_M \sim m_{\text{GUT}}$ (see, *e.g.* Ref. [18] for numerical formulae). All coefficients $c_u, c_d, c_{uR}, c_{dR} < 0$. Hence, the Yukawa corrections reduce the low energy values of the diagonal $(M_{\tilde{q}A}^2)_{33}$ entries with respect to the high energy ones. Note that we neglect subdominant (MFV) terms with higher powers of the Yukawa couplings; the general form of the MFV soft terms is given in Ref. [19].

Before we derive our order of magnitude estimates for the various δ_{ij}^q parameters, two comments are in order:

1. In the following we use the various $\tilde{m}_{ij}^2(m_Z)$ to evaluate the denominator of the $(\delta_{ij}^q)_A$ parameters instead of using the physical mass average as in Section II. In this way we neglect D -terms of $\mathcal{O}(m_Z^2/\tilde{m}_{ij}^2)$ and F -terms of at most $\mathcal{O}(m_t^2/\tilde{m}_{i3}^2)$. It is straightforward to include such corrections into our analysis, but since the flavor pattern from FN gravity is only accurate up to order one numbers, this does not improve the precision of our predictions.

2. Eq. (3.4) is written in the flavor basis. We can read off the δ^q parameters after rotating the squarks by the same transformation that brings the quarks to mass eigenstates, see Eq. (2.2). This rotation does not change the parametric suppression of the X_{ij} terms, and therefore we can still use the estimates (3.2) in the new basis. The rotation can affect the order one coefficients in these terms, but these are unknown anyway.

We now write the low energy values of the entries in the squark mass matrices in the basis where the quark mass matrices and gluino couplings are diagonal. We are interested in models with $r > y_t^2 |V_{ts}|^2$, in which case the gravity-mediated contributions are non-negligible (see below). We can thus neglect all Yukawa couplings except third generation ones. For MFV contributions, we use notations such as V_{td} to denote the actual contributing CKM element. For the non-MFV contributions, where there is uncertainty of order one, we use, for example, the notation V_{13} to represent parametric suppression that is similar to that of V_{ub} or V_{td} . We obtain ($q = U, D, i \neq 3$):

$$\begin{aligned}
(\widetilde{M}_{\tilde{q}_L}^2(m_Z))_{33} &\sim \tilde{m}_{Q_L}^2(r_3 + c_u y_t^2 + c_d y_b^2 + r), \\
(\widetilde{M}_{\tilde{q}_L}^2(m_Z))_{ii} &\sim \tilde{m}_{Q_L}^2(r_3 + r), \\
(\widetilde{M}_{\tilde{U}_L}^2(m_Z))_{12} &\sim \tilde{m}_{Q_L}^2(c_d y_b^2 V_{ub} V_{cb}^* + r |V_{12}|), \\
(\widetilde{M}_{\tilde{U}_L}^2(m_Z))_{i3} &\sim \tilde{m}_{Q_L}^2(c_d y_b^2 V_{ib} V_{tb}^* + r |V_{i3}|), \\
(\widetilde{M}_{\tilde{D}_L}^2(m_Z))_{12} &\sim \tilde{m}_{Q_L}^2(c_u y_t^2 V_{ts} V_{td}^* + r |V_{12}|), \\
(\widetilde{M}_{\tilde{D}_L}^2(m_Z))_{i3} &\sim \tilde{m}_{Q_L}^2(c_u y_t^2 V_{ti} V_{td}^* + r |V_{i3}|).
\end{aligned} \tag{3.7}$$

Hence, with $r \ll r_3$,

$$\begin{aligned}
(\delta_{12}^u)_L &\sim \frac{|V_{12}|}{r_3} \max(r, c_d y_b^2 |V_{ub} V_{cb}^* / V_{12}|) \sim r \frac{|V_{12}|}{r_3}, \\
(\delta_{12}^d)_L &\sim \frac{|V_{12}|}{r_3} \max(r, c_u y_t^2 |V_{ts} V_{td}^* / V_{12}|) \sim r \frac{|V_{12}|}{r_3}, \\
(\delta_{i3}^u)_L &\sim \frac{|V_{i3}|}{r_3} \max(r, c_d y_b^2) \sim \hat{r} \frac{|V_{i3}|}{r_3}, \\
(\delta_{i3}^d)_L &\sim \frac{|V_{i3}|}{r_3} \max(r, c_u y_t^2) \sim \frac{|V_{i3}|}{r_3}, \\
\delta_{i3}^{cu} &\simeq (\delta_{i3}^d)_L,
\end{aligned} \tag{3.8}$$

where

$$\hat{r} \equiv \max\{r, y_b^2\}. \tag{3.9}$$

TABLE II: The order of magnitude estimates for $(\delta_{ij}^{d,u})_{L,R}$ and $\langle \delta_{ij}^{d,u} \rangle$ in the hybrid gauge-gravity models. The numerical estimates are obtained using quark masses at the scale m_Z [20], and taking $r_3 = 3$. All results scale as $(3/r_3)$.

$q \ ij$	$(\delta_{ij}^q)_L$	$(\delta_{ij}^q)_R$	$\langle \delta_{ij}^q \rangle$
$d \ 12$	$(r/r_3) V_{12} \sim 0.08r$	$\frac{(r/r_3)(m_d/m_s)}{ V_{12} } \sim 0.08r$	$(r/r_3)\sqrt{m_d/m_s} \sim 0.08r$
$d \ 13$	$ V_{13} /r_3 \sim 0.001$	$\frac{(r/r_3)(m_d/m_b)}{ V_{13} } \sim 0.08r$	$\sqrt{rm_d/m_b}/r_3 \sim 0.01\sqrt{r}$
$d \ 23$	$ V_{23} /r_3 \sim 0.01$	$\frac{(r/r_3)(m_s/m_b)}{ V_{23} } \sim 0.2r$	$\sqrt{rm_s/m_b}/r_3 \sim 0.05\sqrt{r}$
$u \ 12$	$(r/r_3) V_{12} \sim 0.08r$	$\frac{(r/r_3)(m_u/m_c)}{ V_{12} } \sim 0.003r$	$(r/r_3)\sqrt{m_u/m_c} \sim 0.02r$
$u \ 13$	$(\hat{r}/r_3) V_{13} \sim 0.001\hat{r}$	$\frac{(r/r_3)(m_u/m_t)}{ V_{13} } \sim 0.0006r$	$\sqrt{r\hat{r}m_u/m_t}/r_3 \sim 0.0009\sqrt{r\hat{r}}$
$u \ 23$	$(\hat{r}/r_3) V_{23} \sim 0.01\hat{r}$	$\frac{(r/r_3)(m_c/m_t)}{ V_{23} } \sim 0.03r$	$\sqrt{r\hat{r}m_c/m_t}/r_3 \sim 0.02\sqrt{r\hat{r}}$

Given that $y_b^2 \sim 0.001 \tan^2 \beta$, the distinction between \hat{r} and r is important only if $\tan \beta$ is large.

We can now explain our choice to focus on the region of $r > y_t^2 |V_{ts}|^2$. If r were smaller than that, then MFV contributions would dominate $(\delta_{12}^d)_L$ and, for $\tan \beta \gtrsim 10$, also $(\delta_{12}^u)_L$.

For the $(\delta_{ij}^q)_R$, $q = U, D$ we obtain $i \neq 3$, $j = 1, 2, 3$:

$$\begin{aligned}
(\tilde{M}_{U_R}^2(m_Z))_{33} &\sim \tilde{m}_{U_R}^2(r_3 + c_{uR}y_t^2 + r), \\
(\tilde{M}_{D_R}^2(m_Z))_{33} &\sim \tilde{m}_{D_R}^2(r_3 + c_{dR}y_b^2 + r), \\
(\tilde{M}_{\bar{q}_R}^2(m_Z))_{ii} &\sim \tilde{m}_{\bar{q}_R}^2(r_3 + r), \\
(\tilde{M}_{\bar{q}_R}^2(m_Z))_{ij} &\sim \tilde{m}_{\bar{q}_R}^2 r \frac{m_{q_i}}{m_{q_j} |V_{ij}|},
\end{aligned} \tag{3.10}$$

hence

$$(\delta_{ij}^q)_R \sim \frac{r}{r_3} \frac{m_{q_i}}{m_{q_j} |V_{ij}|}. \tag{3.11}$$

We finally obtain the order of magnitude estimates for the δ_{ij}^q parameters presented in Table II. We would like to emphasize the following points:

1. The RGE suppresses the flavor violating δ^q parameters.
2. The values of $(\delta_{i3}^d)_L$ are independent of r . The reason for this are the RGE-induced $\mathcal{O}(y_t^2)$ terms which dominate the gravity-mediated ones of order r .

3. The values of $\langle \delta_{ij}^q \rangle$ are independent of the CKM parameters.

One of the issues that we are trying to clarify is whether one can differentiate between MFV and non-MFV mediation of supersymmetry breaking. Indeed, our framework gives contributions to $(\delta_{ij}^q)_R$ that cannot be achieved in MFV models. The parameters $(\delta_{ij}^d)_L$, however, receive a contribution from MFV initial conditions (such as pure gauge mediation), which is CKM induced and of the order $(V_{tj}V_{ti}^*/r_3)[y_t^2/(16\pi^2)]\ln(m_M/m_Z)$ times a numerical factor of $\mathcal{O}(5)$ (see Appendix A). For $j = 3$ this is the dominant contribution and, therefore, $(\delta_{i3}^d)_L$ itself is not indicative of hybrid mediation. For $r < y_t^2|V_{ts}|^2$, even the $(\delta_{12}^d)_L$ would be dominated by the MFV contribution. A similar comment applies to $(\delta_{ij}^u)_L$ for large $\tan\beta$ due to the $V_{ib}V_{jb}^*y_b^2$ induced RGE contribution.

C. Splittings and mixing

A flavor changing δ_{ij} parameter depends on three factors: the overall squark mass scale \tilde{m}_{ij} , the mass splitting $\Delta\tilde{m}_{ij}^2$, and the mixing angle K_{ij} . While low energy measurements of FCNC processes are sensitive only to the δ_{ij}^q parameters, high- p_T experiments can, in principle, measure each of these three ingredients separately, hence providing further information regarding the supersymmetric flavor structure [1]. It is thus of interest to estimate $\Delta\tilde{m}_{ij}^2/\tilde{m}_{ij}^2$ and K_{ij} in our hybrid gauge-gravity framework.

Investigation of Eqs. (3.7), (3.10) and the analysis of Appendix A leads to the following estimates of the m_Z -scale mass splittings:

$$\begin{aligned} \frac{\Delta\tilde{m}_{12}^2}{\tilde{m}_{12}^2} &\sim r/r_3 \quad (\tilde{D}_L, \tilde{U}_L, \tilde{D}_R, \tilde{U}_R), \\ \frac{\Delta\tilde{m}_{i3}^2}{\tilde{m}_{i3}^2} &\sim \begin{cases} 1/r_3 \quad (\tilde{D}_L, \tilde{U}_L, \tilde{U}_R) \\ \hat{r}/r_3 \quad (\tilde{D}_R) \end{cases} \quad \text{for } i \neq 3. \end{aligned} \quad (3.12)$$

As concerns the mixing matrices, they depend on the unitary matrices that diagonalize the various quark and squark mass matrices. We define:

$$\begin{aligned} V_L^d m_D V_R^{d\dagger} &= \text{diag}(m_d, m_s, m_b), \\ V_L^u m_U V_R^{u\dagger} &= \text{diag}(m_u, m_c, m_t), \\ \tilde{V}_A^d \overline{M}_{\tilde{D}_A}^2 \tilde{V}_A^{d\dagger} &= \text{diag}(\tilde{m}_{\tilde{d}_{A1}}^2, \tilde{m}_{\tilde{d}_{A2}}^2, \tilde{m}_{\tilde{d}_{A3}}^2), \\ \tilde{V}_A^u \overline{M}_{\tilde{U}_A}^2 \tilde{V}_A^{u\dagger} &= \text{diag}(\tilde{m}_{\tilde{u}_{A1}}^2, \tilde{m}_{\tilde{u}_{A2}}^2, \tilde{m}_{\tilde{u}_{A3}}^2), \end{aligned} \quad (3.13)$$

where $A = L, R$. We obtain for the mixing matrices relevant in neutral gaugino couplings

$$K_A^q = V_A^q \tilde{V}_A^{q\dagger}, \quad (3.14)$$

and for the quark mixing matrix:

$$V = V_L^u V_L^{d\dagger}. \quad (3.15)$$

The parametric suppression of the off-diagonal terms in V_A^q in the FN basis (that is, the basis where the FN charges are well-defined) is determined by the quark flavor parameters:

$$\begin{aligned} (V_L^d)_{ij} &\sim |V_{ij}|, \\ (V_L^u)_{ij} &\sim |V_{ij}|, \\ (V_R^d)_{ij} &\sim \frac{m_{d_i}/m_{d_j}}{|V_{ij}|}, \\ (V_R^u)_{ij} &\sim \frac{m_{u_i}/m_{u_j}}{|V_{ij}|}. \end{aligned} \quad (3.16)$$

The parametric suppression of the off-diagonal terms in \tilde{V}_A^q in the FN basis is determined by r and by the quark flavor parameters:

$$\begin{aligned} (\tilde{V}_L^d)_{12} &\sim |V_{12}|, & (\tilde{V}_L^d)_{i3} &= (V_L^u)_{i3} + \mathcal{O}(\hat{r}|V_{i3}|), \\ (\tilde{V}_L^u)_{12} &\sim |V_{12}|, & (\tilde{V}_L^u)_{i3} &= (V_L^u)_{i3} + \mathcal{O}(\hat{r}|V_{i3}|), \\ (\tilde{V}_R^d)_{12} &\sim \frac{m_d/m_s}{|V_{12}|}, & (\tilde{V}_R^d)_{i3} &= (V_R^d)_{i3} + \mathcal{O}\left(\frac{r(m_{d_i}/m_b)}{\hat{r}|V_{i3}|}\right), \\ (\tilde{V}_R^u)_{12} &\sim \frac{m_u/m_c}{|V_{12}|}, & (\tilde{V}_R^u)_{i3} &= (V_R^u)_{i3} + \mathcal{O}\left(\frac{r(m_{u_i}/m_t)}{|V_{i3}|}\right). \end{aligned} \quad (3.17)$$

We note the following points, which can be further understood on the basis of our analysis in Appendix A:

1. In the up quark mass basis, $(\tilde{V}_L^d)_{i3} \sim (\tilde{V}_L^u)_{i3} \sim \hat{r}|V_{i3}|$. The reason is that in this basis the $Y_u Y_u^\dagger$ term in the RGE is diagonal, and the leading non-diagonal contribution is either the r -suppressed gravity-mediated contribution or the y_b^2 -suppressed MFV contribution.
2. In the up quark mass basis, $(\tilde{V}_R^u)_{i3} \sim r(m_{u_i}/m_t)/|V_{i3}|$. The reason is that in this basis the $Y_u^\dagger Y_u$ term in the RGE is diagonal, and the leading non-diagonal contribution is the r -suppressed gravity-mediated contribution.
3. In the down quark mass basis, $(\tilde{V}_R^d)_{i3} \sim (r/\hat{r})(m_{d_i}/m_b)/|V_{i3}|$. The reason is that in this basis the $Y_d^\dagger Y_d$ term in the RGE is diagonal, and the leading non-diagonal contribution is the r -suppressed gravity-mediated contribution.

We thus find

$$\begin{aligned}
(K_L^d)_{12} &\sim |V_{12}|, & (K_L^d)_{i3} &\sim |V_{ti}|, \\
(K_L^u)_{12} &\sim |V_{12}|, & (K_L^u)_{i3} &\sim \hat{r}|V_{i3}|, \\
(K_R^d)_{12} &\sim \frac{m_d/m_s}{|V_{12}|}, & (K_R^d)_{i3} &\sim \frac{r(m_{d_i}/m_b)}{\hat{r}|V_{i3}|}, \\
(K_R^u)_{12} &\sim \frac{m_u/m_c}{|V_{12}|}, & (K_R^u)_{i3} &\sim \frac{r(m_{u_i}/m_t)}{|V_{i3}|}.
\end{aligned} \tag{3.18}$$

IV. PHENOMENOLOGICAL CONSEQUENCES

By comparing the phenomenological constraints of Table I to the theoretical order of magnitude predictions of the hybrid gauge-gravity models of Table II, we can put an upper bound on r and on \hat{r} , and describe the possible FCNC effects of the model. The strongest bound on r comes from the $\langle \delta_{12}^d \rangle$ parameter, and it reads

$$r/r_3 \lesssim 0.01 - 0.03. \tag{4.1}$$

We use here $m_{\tilde{q}} = 1$ TeV; the bounds would be stronger by $m_{\tilde{q}}/(1 \text{ TeV})$ for lighter $m_{\tilde{q}}$. The stronger bound corresponds to $x = 1$ and a phase of order 0.3, while the weaker bound corresponds to $x = 4$ and a phase smaller than 0.1. The \hat{r} parameter affects only the δ_{i3}^u parameters, so there is no phenomenological constraint on its size, and it is only bounded by its definition:

$$r \leq \hat{r} \lesssim 1. \tag{4.2}$$

For small values of $\tan \beta$, $\hat{r} = r$ and Eq. (4.1) applies to \hat{r} . Inserting $r/r_3 \lesssim 0.03$ and $r \leq \hat{r} \lesssim 1$ into the predictions of Table II, we obtain the upper bounds on the δ_{ij}^q given in Table III.

We then learn that the maximal possible effects in the neutral B_d , B_s and D systems, are as follows (for $r_3 = 3$):

$$\begin{aligned}
B_d : & |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| \lesssim 0.002, \\
B_s : & |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| \lesssim 0.005, \\
D : & |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| \lesssim 0.05.
\end{aligned} \tag{4.3}$$

Note that for D -meson mixing, we use for M_{12}^{exp} the experimental upper bound. The stronger this bound will become, the more significant role the SUSY contribution can play.

TABLE III: The order of magnitude upper bounds on $(\delta_{ij}^{d,u})_{L,R}$ and $\langle \delta_{ij}^{d,u} \rangle$ for $r/r_3 \lesssim 0.03$. Entries in parenthesis are independent of r , therefore representing estimates rather than upper bounds, and scale as $(3/r_3)$. The bounds on $\langle \delta_{13,23}^d \rangle$ scale as $\sqrt{3/r_3}$. The bounds on $(\delta_{i3}^u)_L$ [$\langle \delta_{i3}^u \rangle$] correspond to $\hat{r} \sim 1$ and scale as $(3/r_3)$ [$\sqrt{3/r_3}$]; if $\hat{r} = r$, these bounds are a factor of 10 [$\sqrt{10}$] stronger and do not scale with r_3 .

q	ij	$(\delta_{ij}^q)_L$	$(\delta_{ij}^q)_R$	$\langle \delta_{ij}^q \rangle$
d	12	0.007	0.007	0.007
d	13	[0.001]	0.007	0.003
d	23	[0.01]	0.01	0.01
u	12	0.007	0.0003	0.001
u	13	0.001	0.00005	0.0003
u	23	0.01	0.003	0.006

We emphasize the following points:

1. The bound in the D system comes from $\langle \delta_{12}^u \rangle$ and is r_3 independent.
2. For $r_3 = \mathcal{O}(1 - 10)$, the bound in the B_s system comes from $\langle \delta_{23}^d \rangle$ and scales as $3/r_3$.
3. For $r_3 = \mathcal{O}(1 - 5)$, the bound in the B_d system comes from $\langle \delta_{13}^d \rangle$ and scales as $3/r_3$.
For $r_3 > 5$, the bound comes from $(\delta_{13}^d)_R$ and does not scale with r_3 .

For large $\tan \beta$ and low M_{A^0} , the $B_{d,s}$ mixing amplitudes can be significantly enhanced, as discussed in Section II C. Comparing the phenomenological constraints of Eq. (2.14) to Table II, we obtain for $r_3 = 3$ (and $\tan \beta = 30$, $M_{A^0} = 200$ GeV):

$$\begin{aligned}
B_d : |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| &\lesssim 0.10, \\
B_s : |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| &\lesssim 0.13.
\end{aligned}
\tag{4.4}$$

The r_3 dependence of upper bounds on the supersymmetric contributions to B_d and B_s mixings is shown in Fig. 1.

We now discuss where further signals of this non-MFV scenario could arise. While the bounds from Eq. (2.13) and (2.14) can be evaded for suitable values of M_{A^0} and $\tan \beta$, they indicate on the other hand that in FN gravity models an observation of $B_{d,s} \rightarrow \mu^+ \mu^-$ decays

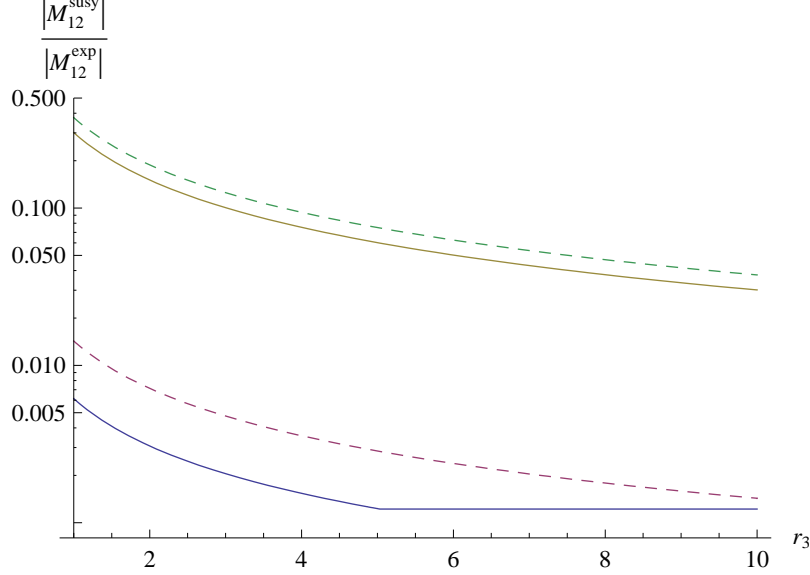


FIG. 1: Maximum reach in B_d (solid) and B_s (dashed) mixing, $|M_{12}^{\text{susy}}/M_{12}^{\text{exp}}|$, as a function of the RGE-factor r_3 . The uppermost two curves correspond to $\tan\beta = 30$ and $M_{A^0} = 200$ GeV.

is possible near their current experimental limits. This itself is, however, not a unique sign of our model, since it can happen also in the MFV MSSM at large $\tan\beta$, e.g., [21]. One crucial difference is the breakdown of MFV relations between $b \rightarrow s$ and $b \rightarrow d$, such as in $B \rightarrow \mu^+\mu^-$ decays [22]. We find for the ratio $R_{\mu\mu}$

$$R_{\mu\mu} = \frac{\mathcal{B}(B_s \rightarrow \mu^+\mu^-)}{\mathcal{B}(B_d \rightarrow \mu^+\mu^-)} \sim \frac{m_{B_s} f_{B_s}^2 \tau_{B_s}}{m_{B_d} f_{B_d}^2 \tau_{B_d}} \times r_{\text{ps}} \times \begin{cases} \frac{|V_{ts}|^2}{|V_{td}|^2} & \text{for } (\text{MFV}, (\delta_{i3}^d)_L), \\ \frac{|m_s V_{td}|^2}{|m_d V_{ts}|^2} & \text{for } ((\delta_{i3}^d)_R), \\ \frac{m_s}{m_d} & \text{for } (\langle \delta_{i3}^d \rangle), \end{cases} \quad (4.5)$$

where f_{B_q} , m_{B_q} and τ_{B_q} denote the decay constant, mass and lifetime of the B_q , $q = d, s$, respectively, and r_{ps} collects all further, small (known) U-spin breaking of $R_{\mu\mu}$ related to kinematical factors.

While stemming from qualitatively very different expressions, numerically the three ratios in Eq. (4.5) turn out to be similar, that is (from top to bottom), 25, 14 and 19, using central values at m_Z from [20]. Since we cannot distinguish the case with dominant $(\delta_{i3}^d)_L$ from MFV, some contribution from $(\delta_{i3}^d)_R$ is required to identify non-MFV. If this is the case, $R_{\mu\mu}$ is suppressed w.r.t. its MFV (and Standard Model) value. Since there is no large hierarchy between $R_{\mu\mu}$ in the different scenarios, establishing the FN flavor quantum numbers in this observable needs a measurement at the $\mathcal{O}(10\%)$ level (3σ) and very good control over f_{B_s}/f_{B_d} .

We close with some general comments. Signals of a FN gravity contribution are those of non-MFV models, that is, *e.g.*, [23], (i) beyond CKM CP-violation, (ii) wrong chirality contributions to FCNCs, and (iii) the breakdown of CKM-relations as in $R_{\mu\mu}$. Because the FN gravity model contains only a controlled amount of flavor violation, an experimental verification needs precise measurements.

Since in FN gravity $(\delta_{i3}^d)_R \gtrsim (\delta_{i3}^d)_L$, see Table III, the natural place to look for such contributions is in right-handed currents. The sensitivity will be even higher if one looks in addition for CP-violation. Potentially interesting here are CP asymmetries in $B \rightarrow K^*(\rightarrow K\pi)\ell^+\ell^-$ decays [24].

The impact of charged wino loops to b -physics observables is limited by (α_2/α_3) with respect to the impact of $(\delta_{i3}^d)_L$, see Eq. (3.8), and is hence sub-dominant. Charged higgsino effects could be of interest at large $\tan\beta$. Further study is needed.

Note that there is also the possibility of a light stop having a macroscopic lifetime of order picoseconds, if the FCNC decay of \tilde{t}_1 to charm plus the lightest neutralino induced by δ_{23}^u is sufficiently suppressed yet is the dominant decay mode [25]. The latter can be arranged kinematically by a small mass splitting, ΔM , between the \tilde{t}_1 and the lightest neutralino. In FN gravity a long-lived stop requires the lightest stop to be predominantly left-handed and $\tan\beta$ to be small, such that $(\delta_{23}^u)_L \lesssim 10^{-6} (m_{\tilde{t}_1}/\Delta M)$. This gives an upper bound $r/r_3 \lesssim 3 \cdot 10^{-4}$ for $\Delta M/m_{\tilde{t}_1} = 0.1$, stronger than the one in Eq. (4.1).

V. HOLOMORPHIC ZEROS

With a more complicated model employing the Froggatt-Nielsen mechanism, one can suppress the supersymmetric mixing angles compared to the values given in Eqs. (3.16) and (3.17), while keeping the parametric suppression of the quark masses and of the CKM angles consistent with the measured values [6]. The horizontal symmetry has to be extended to, for example, $U(1)_1 \times U(1)_2$, and holomorphic zeros must play a role. At least one of the two horizontal $U(1)$'s is broken by a single spurion, and some of the Yukawa couplings carry charge of the same sign as the spurion, and thus are forbidden by holomorphy.

Originally, this mechanism was used to obtain phenomenologically viable models without any squark degeneracy. However, recent improvements in the bound on the mass splitting

in the neutral D system imply that degeneracy between the first two generations of squark doublets at the level of $\mathcal{O}(10\%)$ or stronger is required (for squarks lighter than TeV) [1, 10, 26].

Thus, in this section, we investigate the possibility of constructing such FN-type models, where the required minimal degeneracy comes from either the gauge-mediation dominance or RGE or both. In particular, we ask what are the maximal possible effects in the neutral D, B_d and B_s systems in such a framework.

It was proven in Ref. [3] that, to obtain

$$(K_L^d)_{12} \ll |V_{12}|, \quad (K_R^d)_{12} \ll \frac{m_d/m_s}{|V_{12}|}, \quad (5.1)$$

(as necessary to relax the strong degeneracy requirement), while keeping the CKM elements large enough, there should be four (and only four) specific holomorphic zeros in the down quark mass matrix, leading to both lower and upper bounds on the supersymmetric mixing angles. These bounds are given in Table IV. The parameter ϵ_{\max} stands for the largest among the spurions that break the horizontal FN symmetry. As before, for MFV contributions (namely those that survive in the $r = 0$ limit) we use the notation V_{ti} rather than V_{i3} . The $(K_L^d)_{i3}$ angles get comparable contributions from MFV and non-MFV sources, so we use the V_{i3} notations for these.

The analysis of the $(K_L^d)_{12}$ requires some explanation. The $\tilde{d}_L - \tilde{s}_L$ block of $\tilde{M}_{\tilde{D}_L}^2(m_Z)$ has the following form:

$$\tilde{M}_{\tilde{D}_L}^2(m_Z) \sim \tilde{m}_{\tilde{D}_L}^2 \begin{pmatrix} r_3 + rX_{11} & c_u y_t^2 V_{td}^* V_{ts} + rX_{12} \\ c_u y_t^2 V_{td} V_{ts}^* + rX_{12}^* & r_3 + rX_{22} + c_u y_t^2 |V_{ts}|^2 \end{pmatrix}. \quad (5.2)$$

Here X_{11} and X_{22} are $\mathcal{O}(1)$ and different from each other, while X_{12} is taken to lie in the range $(0, |V_{12}|\epsilon_{\max}^2)$. We remind the reader that we restrict our analysis to the region where r is larger than $y_t^2 |V_{ts}|^2$, so the latter term can be neglected in the (2,2) entry. The lower bound on $(K_L^d)_{12}$ corresponds to a negligibly small X_{12} . The upper bound given in the table corresponds to $|X_{12}| \sim |V_{12}|\epsilon_{\max}^2$ and $r \gtrsim 0.05$. For $r \lesssim 0.05$ it should be replaced with $|V_{td}V_{ts}|/r$.

The δ_{ij}^q parameters are further suppressed by the mass splittings as in Eq. (3.12). Comparing this to Table I, we find that the strongest constraint on r/r_3 comes from the bound on $\langle \delta_{12}^u \rangle$. We obtain

$$r/r_3 \lesssim 0.13, \quad (5.3)$$

TABLE IV: Bounds on the supersymmetric mixing angles in models of alignment with suppressed $(K_{L,R}^d)_{12}$. For the numerical estimates we use quark masses at the scale m_Z [20] and take $r \leq \hat{r} \lesssim 1$, and $\epsilon_{\max} \sim 0.2$.

Mixing angle	Lower bound	Upper bound
$(K_L^d)_{12}$	$ V_{td}V_{ts} /r \sim 0.0005/r$	$ V_{12} \epsilon_{\max}^2 \sim 0.009$
$(K_R^d)_{12}$	$\frac{m_d}{m_s} V_{13}V_{23} \sim 9 \cdot 10^{-6}$	$\frac{m_d}{m_s V_{12} }\epsilon_{\max}^2 \sim 0.009$
$(K_L^d)_{13}$	$ V_{13} \sim 0.004$	$ V_{13} \sim 0.004$
$(K_R^d)_{13}$	$\frac{m_d}{m_b} V_{13} \sim 4 \cdot 10^{-6}$	$\frac{r}{\hat{r}} \frac{m_d}{m_b V_{13} }\epsilon_{\max}^2 \sim 0.009$
$(K_L^d)_{23}$	$ V_{23} \sim 0.04$	$ V_{23} \sim 0.04$
$(K_R^d)_{23}$	$\frac{m_s}{m_b} V_{23} \sim 0.0008$	$\frac{r}{\hat{r}} \frac{m_s}{m_b V_{23} }\epsilon_{\max}^2 \sim 0.02$
$(K_L^u)_{12}$	$ V_{12} \sim 0.2$	$ V_{12} \sim 0.2$
$(K_R^u)_{12}$	$\frac{m_u}{m_c} V_{12} \sim 0.0005$	$\frac{m_u}{m_c V_{12} } \sim 0.009$

TABLE V: Upper bounds on the parametric suppression of $(\delta_{ij}^{d,u})_{L,R}$ and $\langle \delta_{ij}^{d,u} \rangle$ in the hybrid gauge-gravity models with alignment and suppressed δ_{12}^d . For the numerical evaluation we take $r/r_3 \sim 0.13$, $r \leq \hat{r} \lesssim 1$ and $r_3 = 3$. $(\delta_{13,23}^d)_L$ scale as $(3/r_3)$, and $\langle \delta_{13,23}^d \rangle$ scale as $\sqrt{3/r_3}$.

$q \ ij$	$(\delta_{ij}^q)_L$	$(\delta_{ij}^q)_R$	$\langle \delta_{ij}^q \rangle$
$d \ 12$	$(r/r_3) V_{12} \epsilon_{\max}^2 \sim 0.001$	$\frac{(r/r_3)m_d\epsilon_{\max}^2}{ V_{12} m_s} \sim 0.001$	$(r/r_3)\sqrt{m_d/m_s}\epsilon_{\max}^2 \sim 0.001$
$d \ 13$	$ V_{13} /r_3 \sim 0.001$	$\frac{(r/r_3)m_d\epsilon_{\max}^2}{ V_{13} m_b} \sim 0.001$	$\sqrt{r(m_d/m_b)}\epsilon_{\max}/r_3 \sim 0.001$
$d \ 23$	$ V_{23} /r_3 \sim 0.01$	$\frac{(r/r_3)m_s\epsilon_{\max}^2}{ V_{23} m_b} \sim 0.002$	$\sqrt{r(m_s/m_b)}\epsilon_{\max}/r_3 \sim 0.006$
$u \ 12$	$(r/r_3) V_{12} \sim 0.03$	$\frac{(r/r_3)m_u}{ V_{12} m_c} \sim 0.001$	$(r/r_3)\sqrt{m_u/m_c} \sim 0.006$

in agreement with previous works [1, 10, 26]. Estimates for all δ_{ij}^q parameters are given in Table V (for $r_3 = 3$).

We then learn that, in the case that holomorphic zeros play a role in making the alignment accurate so that the degeneracy is weakest, the maximal possible effects in the neutral B_d , B_s and D systems, are as follows (for $r_3 = 3$):

$$\begin{aligned}
B_d : |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| &\lesssim 0.0004, \\
B_s : |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| &\lesssim 0.0008, \\
D : |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| &\lesssim 1.
\end{aligned}
\tag{5.4}$$

Thus, the precise alignment further suppresses the new physics effect in the B_d and B_s mixings. On the other hand, since – by construction – it does not affect the up sector, the milder degeneracy allows large (and possibly CP violating) effects in the neutral D system.

VI. PROBING MESSENGERS

We now ask what the constraints derived from FCNC processes, specifically the upper bound on r/r_3 given in Eq. (4.1), imply for the parameters of gauge mediation.

Given the soft parameters at the high scale, the RGE-factor r_3 defined via Eq. (3.5) is calculable from the MSSM running of the soft squark masses, the one loop running of which is also discussed in Appendix A. Neglecting contributions from the electroweak gauge couplings, one obtains an analytical expression for r_3 (see, *e.g.*, [27]):

$$r_3 = r_3(m_M) = 1 + \frac{8}{3\pi} \left(\int_{\ln(m_Z)}^{\ln(m_M)} dt \frac{\alpha_3^3(t)}{\alpha_3^2(m_M)} \right) \frac{M_3^2(m_M)}{\tilde{m}_{12}^2(m_M)}. \tag{6.1}$$

Here, M_3 denotes the gluino mass and \tilde{m}_{12}^2 is defined in Eq. (3.6). In messenger models of gauge mediation, the ratio M_3^2/\tilde{m}_{12}^2 is determined by a simple formula at the scale of mediation:

$$\frac{M_3^2(m_M)}{\tilde{m}_{12}^2(m_M)} = \frac{3}{8} N_M + \mathcal{O} \left[\left(\frac{\alpha_i}{\alpha_3} \right)^2 \right], \quad i = 1, 2, \quad \text{for } q = Q_L, U_R, D_R, \tag{6.2}$$

where N_M denotes the number of color-triplet messengers. We explicitly see that in our approximation, due to the universality of the initial conditions and the running, r_3 is universal for Q_L, U_R and D_R soft masses. We depict r_3 as a function of the messenger scale for $N_M = 1$ and $N_M = 3$ in Fig. 2. It depends logarithmically on m_M , and grows with N_M .

The parameter r introduced in the initial conditions of gauge-gravity models at the messenger scale m_M , Eq. (3.1), can be expressed as a ratio of soft squark masses:

$$r = \frac{\tilde{m}_{12\text{-gravity}}^2}{\tilde{m}_{12\text{-gauge}}^2} \sim \left(\frac{m_M}{m_{\text{Pl}}} \right)^2 \left(\frac{4\pi}{\alpha_3(m_M)} \right)^2 \frac{3}{8} \frac{1}{N_M}, \tag{6.3}$$

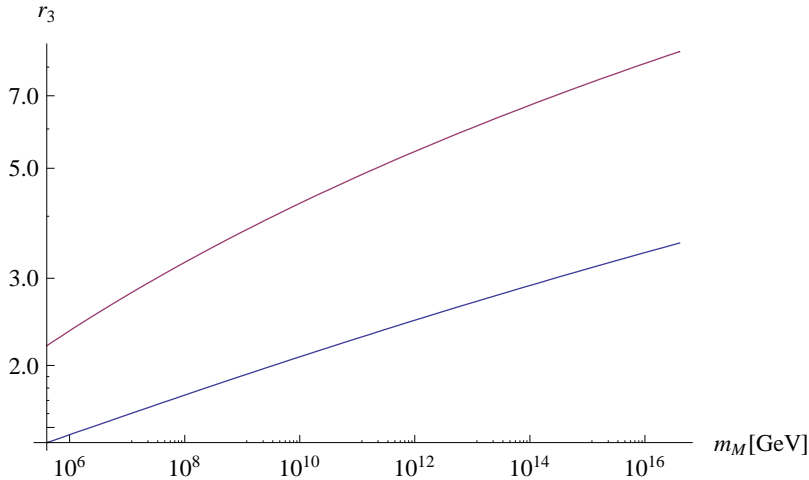


FIG. 2: The RGE-factor r_3 as a function of the messenger scale for $N_M = 1$ (lower curve) and $N_M = 3$ (upper curve).

where $m_{\text{Pl}} \sim 10^{19}$ GeV denotes the Planck mass. In Eq. (6.3) we again neglect contributions other than from the strong interaction as well as running of the gravity-induced soft terms above m_M .

Eq. (4.1) implies the existence of an upper bound on the messenger scale or, in other words, a minimal separation between the scales of gravity- and gauge-mediation. We find that flavor physics determines this to be about three orders of magnitude, *i.e.*, $m_M \lesssim m_{\text{Pl}}/10^3$. A larger number of messengers gives a heavier spectrum, and hence a weaker bound. This is also illustrated in Fig. 3.

In writing Eq. (6.3) we assumed that the highest F -term contributes to gauge mediation. If this is not the case, r gets enhanced by $\langle F \rangle^2 / \langle F_M \rangle^2$, the square of the ratio of the highest F -term vev to the one that couples to the messengers. The flavor constraint Eq. (4.1) requires then a low m_M , or, turning the argument around, indicates gravity-mediated contributions can be non-negligible even if the scale of gauge mediation is low.

It has been pointed out recently that hidden sector effects modify in general the initial conditions below which the known MSSM-RG equations apply [28]. If the hidden sector is weakly interacting, then the effects are small and our analysis holds to this degree. If the renormalization is non-perturbative, our analysis will depend on the unknown hidden sector physics. A general framework, termed general gauge mediation, to account for this has been outlined in [29].

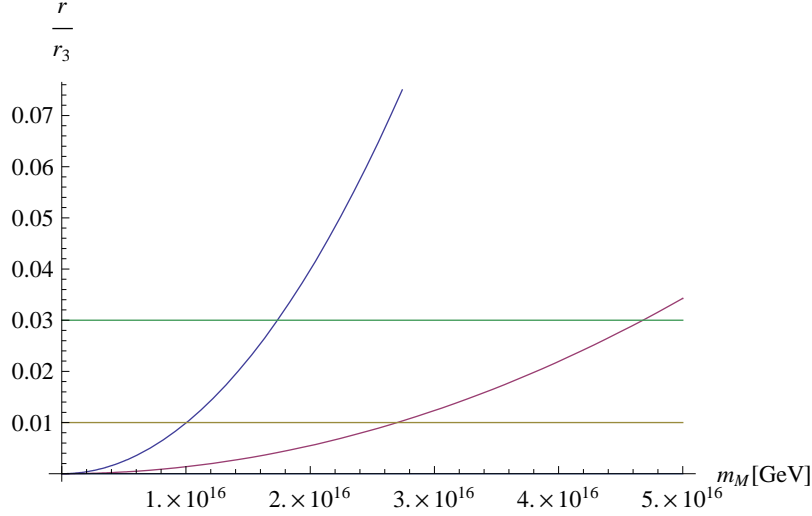


FIG. 3: r/r_3 as a function of the messenger scale for $N_M = 1$ (upper curve) and $N_M = 3$ (lower curve) from Eq. (6.3). The horizontal lines correspond to the FCNC upper bounds of Eq. (4.1).

Within general gauge mediation, our analysis is affected in the following ways:

1. The relation between the gluino mass and the soft squark masses, Eq. (6.2), can receive order one corrections. The outcome of this for the example of a change of factor three in the initial conditions is illustrated by the difference in the curves of Fig. 2. In other words, we cannot calculate r_3 without knowledge of the hidden sector.
2. The initial conditions for the soft squark masses, Eq. (3.1), are not of perturbative messenger gauge-mediation type. In particular, the soft masses for Q_L, U_R and D_R are renormalized differently, and in general we need to introduce several RGE-parameters r_3 . Note that, as in the minimal case, in the limit of $\alpha_1, \alpha_2 \rightarrow 0$ we recover universality of soft masses and hence, of r_3 . Since the corrections arise in full generality non-perturbatively, this might not be representing the true spectrum.
3. Unlike perturbative messenger mediation, general gauge mediation does not exclude $\tilde{m}_{12}^2(m_M) < 0$. Consequently, $r_3 < 1$ becomes possible, see Eq. (6.1). To avoid a tachyonic spectrum, then, however, a very large RGE effect is required such that $r_3 < 0$.
4. We cannot express r in terms of messenger parameters as simply as Eq. (6.3).

What, however, still remains valid in general gauge mediation is the form of Eq. (3.1).

In particular, the hidden sector effects do not introduce further flavor violation into the soft masses because gauge mediation respects the $U(3)^5$ global flavor symmetry.

By not fixing r_3 to a specific, minimal gauge-mediation value, we have hence mimicked hidden sector effects in Section IV.

VII. CONCLUSIONS

We considered supersymmetric models where squark masses are dominated by gauge-mediated contributions, yet gravity-mediated contributions are not negligible. Such a situation arises when the messenger scale is not much below $\alpha_3 m_{\text{Pl}}$, or when the F -term that leads to gauge mediation is at a scale much lower than the highest F -term. We further assumed that the gravity-mediated contributions follow selection rules that arise from a Froggatt-Nielsen symmetry that explains the hierarchy in the Yukawa couplings. Such models constitute an example of viable and natural supersymmetric models that are not minimally flavor violating (non-MFV). The mass splittings and flavor decomposition of sfermions can perhaps be directly measured in the ATLAS/CMS experiments [1].

We posed here the question of whether measurements of FCNC processes, such as neutral meson mixing, can show signals of such non-MFV models. We found that the strongest bound on the mass splitting between the first two squark generations $\Delta\tilde{m}_{12}^2/\tilde{m}_{12}^2$ comes from $K^0 - \overline{K}^0$ mixing, and is of $\mathcal{O}(0.03)$. This splitting reflects the relative size of the gravity- and gauge-mediated contributions which, at the mediation scale, gets lifted by an inverse RGE-factor w.r.t. the physical splitting at the electroweak scale. We obtain for the respective splitting at the mediation scale a value that is constrained to be below $\mathcal{O}(0.1)$ for minimal gauge mediation with one messenger, or even as large as $\mathcal{O}(0.3)$ in general gauge mediation, or with several messengers.

The slepton sector has also been studied within hybrid gauge-gravity mediation [1]. Assuming the simplest FN charge assignments, no parametric suppression of the 1-2 lepton mixing angle and minimal gauge mediation giving sleptons lighter than squarks, the bounds from lepton flavor changing processes on the splittings are stronger than those from the quarks.

Given the constraint on the splitting, the order of magnitude predictions that follow from the FN symmetry, and the RGE effects, we evaluated the maximal possible modifications

to the Standard Model predictions to various FCNC processes. We found that the effects on the $B_d - \overline{B}_d$ and $B_s - \overline{B}_s$ mixing amplitudes is generically below the percent level, but can be of order ten percent for large $\tan\beta$. It is maximized when the RGE suppression is minimal.

On the other hand, the effect on the $D^0 - \overline{D}^0$ mixing amplitude can be $\mathcal{O}(1)$ (and CP violating), though in the simplest models it is at most of order five percent. We found also that the ratio of $B_s \rightarrow \mu^+\mu^-$ to $B_d \rightarrow \mu^+\mu^-$ branching ratios is sensitive to the FN flavor symmetries.

Further possibilities to test FN gravity, that is, Planck scale physics, with rare decays are pointed out. Particularly promising are searches for right-handed currents, if possible even in conjunction with CP-violation.

When thinking about the future of experimental flavor physics, and evaluating the sensitivity to new physics of, for example, a super-B factory [11, 31], a question that often arises is the following: What experimental accuracy is worth achieving, given well-motivated models of new physics as well as theoretical (QCD-related) uncertainties. Eqs. (4.3), (4.4) and (5.4) provide a concrete answer – within a specific but well-motivated and natural framework – to this question. An accuracy of order a few percent in measurements related to neutral D , B_d or B_s mixing may be sensitive to new physics. Since the new physics that we discuss introduces, in general, new CP violating phases of order one, a theoretically clean signal for the new physics can be established by measuring CP asymmetries at that level.

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APPENDIX A: RGE EFFECTS

In the Appendix, we present the renormalization group equations for the quark and squark parameters relevant to our framework. (General formulae are given in Ref. [30].)

We use the following approximations:

1. We neglect the RGE effects of the first and second generation Yukawa couplings y_u, y_d, y_s and y_c .
2. We neglect the RGE effects that involve $|V_{ts}|^2, |V_{td}|^2$ and $V_{td}V_{ts}^*$.
3. We neglect the effects of the off-diagonal elements in the squark mass-squared matrices on the running of the diagonal terms.

(Within the special class of models discussed in Section V, some of these approximations are not valid, and then we do include the relevant factors.)

We obtain for the CKM mixing angles [32]

$$16\pi^2 \frac{d}{dt} \ln V_{\alpha\beta} = \begin{cases} -y_t^2 - y_b^2 & \text{for } V_{ub}, V_{cb}, V_{td}, V_{ts} \\ 0 & \text{for } V_{ud}, V_{us}, V_{cd}, V_{cs}, V_{tb} \end{cases} \quad (\text{A1})$$

and for the Yukawa coupling ratios (or, equivalently, mass ratios)

$$\begin{aligned} 16\pi^2 \frac{d}{dt} \ln(y_u/y_c) &= 0, \\ 16\pi^2 \frac{d}{dt} \ln(y_c/y_t) &= -3y_t^2 - y_b^2, \\ 16\pi^2 \frac{d}{dt} \ln(y_d/y_s) &= 0, \\ 16\pi^2 \frac{d}{dt} \ln(y_s/y_b) &= -y_t^2 - 3y_b^2, \\ 16\pi^2 \frac{d}{dt} \ln[V_{cb}/(y_c/y_t)] &= 2y_t^2, \\ 16\pi^2 \frac{d}{dt} \ln[V_{cb}/(y_s/y_b)] &= 2y_b^2. \end{aligned} \quad (\text{A2})$$

For the diagonal elements in the soft squark mass-squared matrices, we obtain ($i = 1, 2, 3$)

$$\begin{aligned} 16\pi^2 \frac{d}{dt} (M_{\tilde{Q}_L}^2)_{ii} &= 2[(M_{\tilde{Q}_L}^2)_{33} + (M_{\tilde{U}_R}^2)_{33} + m_{H_u}^2] y_t^2 \delta_{i3} \\ &\quad + 2[(M_{\tilde{Q}_L}^2)_{33} + (M_{\tilde{D}_R}^2)_{33} + m_{H_d}^2] y_b^2 \delta_{i3} - \frac{32}{3} g_3^2 |M_3|^2 + \mathcal{O}(g_2^2, g_1^2), \\ 16\pi^2 \frac{d}{dt} (M_{\tilde{U}_R}^2)_{ii} &= 4[(M_{\tilde{U}_R}^2)_{33} + (M_{\tilde{Q}_L}^2)_{33} + m_{H_u}^2] y_t^2 \delta_{i3} - \frac{32}{3} g_3^2 |M_3|^2 + \mathcal{O}(g_2^2, g_1^2), \\ 16\pi^2 \frac{d}{dt} (M_{\tilde{D}_R}^2)_{ii} &= 4[(M_{\tilde{D}_R}^2)_{33} + (M_{\tilde{Q}_L}^2)_{33} + m_{H_d}^2] y_b^2 \delta_{i3} - \frac{32}{3} g_3^2 |M_3|^2 + \mathcal{O}(g_2^2, g_1^2). \end{aligned} \quad (\text{A3})$$

For the off-diagonal terms involving the third generation, we obtain, in the super-CKM basis (where gluino couplings and quark masses are diagonal), ($i \neq 3$)

$$\begin{aligned}
16\pi^2 \frac{d}{dt} (\widetilde{M}_{\tilde{U}_L}^2)_{i3} &= [(M_{\tilde{Q}_L}^2)_{ii} + (M_{\tilde{Q}_L}^2)_{33} + 2(M_{\tilde{D}_R}^2)_{33} + 2m_{H_d}^2] y_b^2 V_{ib} V_{tb}^* + (y_t^2 + y_b^2) (\widetilde{M}_{\tilde{U}_L}^2)_{i3} \\
16\pi^2 \frac{d}{dt} (\widetilde{M}_{\tilde{D}_L}^2)_{i3} &= [(M_{\tilde{Q}_L}^2)_{ii} + (M_{\tilde{Q}_L}^2)_{33} + 2(M_{\tilde{U}_R}^2)_{33} + 2m_{H_u}^2] y_t^2 V_{ti}^* V_{tb} + (y_t^2 + y_b^2) (\widetilde{M}_{\tilde{D}_L}^2)_{i3} \\
16\pi^2 \frac{d}{dt} (\widetilde{M}_{\tilde{U}_R}^2)_{i3} &= 2y_t^2 (\widetilde{M}_{\tilde{U}_R}^2)_{i3}, \\
16\pi^2 \frac{d}{dt} (\widetilde{M}_{\tilde{D}_R}^2)_{i3} &= 2y_b^2 (\widetilde{M}_{\tilde{D}_R}^2)_{i3}.
\end{aligned} \tag{A4}$$

The 1 – 2 terms are, within our approximations, RGE invariant:

$$16\pi^2 \frac{d}{dt} (M_{\tilde{Q}_L, \tilde{U}_R, \tilde{D}_R}^2)_{12} = 0. \tag{A5}$$

1. $(\delta_{12}^q)_A$

With our approximations (which hold much more generally than within our specific framework), almost all parameters related to just the first two generations, and, in particular,

$$(M_{\tilde{q}_A}^2)_{12}, \quad (M_{\tilde{q}_A}^2)_{22} - (M_{\tilde{q}_A}^2)_{11}, \tag{A6}$$

are RGE invariant. In models (as ours) where $|(M_{\tilde{q}_A}^2)_{12}| \ll |(M_{\tilde{q}_A}^2)_{22} - (M_{\tilde{q}_A}^2)_{11}|$, Eq. (A6) further implies the RGE invariance of

$$(\tilde{V}_A^q)_{12}, \quad \Delta \tilde{m}_{q_{A2} q_{A1}}^2. \tag{A7}$$

The only parameter related to the first two generations which is not RGE invariant is the average squark mass. The universal QCD effect on the running of the diagonal mass-squared terms is actually the only RGE effect that (for running from high scale, as in our framework) can be significantly larger than one. This is taken into account by the factor r_3 defined in Eq. (3.5). Numerical values within gauge mediation are discussed in Section VI.

The parameters of interest for our purposes are the $(\delta_{ij}^q)_A$ parameters. We analyze the RGE implications on these parameters using the two generation approximation of Eq. (2.5). From Eqs. (A2, A7, 3.5) we learn that

$$(\delta_{12}^q)_A(\mu = m_Z) = \frac{1}{r_3} (\delta_{12}^q)_A(\mu = m_M). \tag{A8}$$

Within our framework, where the structures of the quark and squark mass matrices are related by the FN symmetry, this leads to the values of the $(\delta_{12}^q)_L$ as given in Eq. (3.8) and $(\delta_{12}^q)_R$ as given in Eq. (3.11).

2. $(\delta_{i3}^q)_R$

Within our approximation, we also find from Eqs. (A2) and (A4) that the following two combinations of squark and quark parameters are RGE invariant:

$$\frac{(\widetilde{M}_{\tilde{U}_R}^2)_{i3}}{(y_{u_i}/y_t)/|V_{cb}|}, \quad \frac{(\widetilde{M}_{\tilde{D}_R}^2)_{i3}}{(y_{d_i}/y_b)/|V_{cb}|} \quad (i = 1, 2). \quad (\text{A9})$$

The RGE effects on the splittings are as follows (see Eq. (A3)):

$$\begin{aligned} [(M_{\tilde{U}_R}^2)_{33} - (M_{\tilde{U}_R}^2)_{ii}](\mu = m_Z) &\sim \tilde{m}_q^2, \\ [(M_{\tilde{D}_R}^2)_{33} - (M_{\tilde{D}_R}^2)_{ii}](\mu = m_Z) &\sim \hat{r}\tilde{m}_q^2. \end{aligned} \quad (\text{A10})$$

These equations lead to the estimates of $(\tilde{V}_R^q)_{i3}$ given in Eq. (3.17), $(K_R^q)_{i3}$ as given in Eq. (3.18), and $(\delta_{i3}^q)_R$ as given in Eq. (3.11).

3. $(\delta_{i3}^q)_L$

The situation regarding $(\delta_{i3}^q)_L$ is less simple than the other cases. Here, Eqs. (A1) and (A4) imply, unlike the analogous case for \tilde{q}_R (see Eq. (A9)), that $(M_{\tilde{Q}_L}^2)_{i3}/|V_{ib}|$ is not RGE invariant. Consider first the \tilde{U}_L sector, and assume for simplicity small $\tan\beta$ (so that the y_b^2 -dependent terms in Eq. (A4) can be neglected):

$$16\pi^2 \frac{d}{dt} \ln \frac{(M_{\tilde{U}_L}^2)_{i3}}{|V_{ib}|} = 2(y_t^2 + y_b^2). \quad (\text{A11})$$

For the relevant mass-squared difference, we obtain from Eq. (A3):

$$\begin{aligned} 16\pi^2 \frac{d}{dt} [(M_{\tilde{Q}_L}^2)_{33} - (M_{\tilde{Q}_L}^2)_{ii}] &= 2[(M_{\tilde{Q}_L}^2)_{33} + (M_{\tilde{U}_R}^2)_{33} + m_{H_u}^2]y_t^2 \\ &\quad + 2[(M_{\tilde{Q}_L}^2)_{33} + (M_{\tilde{D}_R}^2)_{33} + m_{H_d}^2]y_b^2. \end{aligned} \quad (\text{A12})$$

The conclusion is that the RGE effects on both $(\tilde{V}_L^u)_{i3}$ and on $|V_{ib}|$ are $\mathcal{O}(1)$ and different from each other. Yet, at low energy, in the up quark mass basis, we have (recall Eq. (3.17) is in the FN basis)

$$|(\tilde{V}_L^u)_{i3}| \sim r|V_{ib}|. \quad (\text{A13})$$

When the MFV y_b^2 dependent terms are taken into account, we obtain Eq. (3.17) for $(\tilde{V}_L^u)_{i3}$, Eq. (3.18) for $(K_L^u)_{i3}$, and Eq. (3.8) for $(\delta_{i3}^u)_L$.

Next consider the running of $(M_{\tilde{D}_L}^2)_{i3}$ in the down quark mass basis. The second term on the right hand side of the relevant Eq. (A4) is smaller by a factor of $\mathcal{O}(r)$ than the first and so $|(\tilde{V}_L^d)_{i3}| \approx |V_{ti}|$. Eqs. (3.18) for $(K_L^d)_{i3}$, and (3.8) for $(\delta_{i3}^d)_L$ follow.

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